

## Evolution of Cool Close Binaries – Rapid Mass Transfer and Near Contact Binaries

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### ABSTRACT

We test the evolutionary model of cool close binaries developed by one of us (KS) on the observed properties of near contact binaries (NCBs). These are binaries with one component filling the inner critical Roche lobe and the other almost filling it. Those with a more massive component filling the Roche lobe are SD1 binaries whereas in SD2 binaries the Roche lobe filling component is less massive. Our evolutionary model assumes that, following the Roche lobe overflow by the more massive component (donor), mass transfer occurs until mass ratio reversal. A binary in an initial phase of mass transfer, before mass equalization, is identified with SD1 binary. We show that the transferred mass forms an equatorial bulge around the less massive component (accretor). Its presence slows down the mass transfer rate to the value determined by the thermal time scale of the accretor, once the bulge sticks out above the Roche lobe. It means, that in a binary with a (typical) mass ratio of 0.5 the SD1 phase lasts at least 10 times longer than resulting from the standard evolutionary computations neglecting this effect. This is why we observe so many SD1 binaries. Our explanation is in contradiction to predictions identifying the SD1 phase with a broken contact phase of the Thermal Relaxation Oscillations model. The continued mass transfer, past mass equalization, results in mass ratio reversed. SD2 binaries are identified with this phase. Our model predicts that the time scales of SD1 and SD2 phases are comparable to one another.

Analysis of the observations of 22 SD1 binaries, 27 SD2 binaries and 110 contact binaries (CBs) shows that relative number of both types of NCBs favors similar time scales of both phases of mass transfer. Total masses, orbital angular momenta and orbital periods of SD1 and SD2 binaries are indistinguishable from each other whereas they differ substantially from the corresponding parameters of CBs. We conclude that the results of the analysis fully support the model presented in this paper.

**Key words:** *Stars: activity – binaries: close – Stars: evolution – Stars: late-type – Stars: rotation*

### 1. Introduction

Near contact binaries (NCBs) are a class of cool close binaries showing eclipses, with one component filling the inner critical equipotential surface (Roche lobe) and the other almost filling it. The name was suggested by Shaw (1990) who dis-

tinguished two types of NCBs: those with the more massive component filling the Roche lobe were named V1010 Oph type and binaries with the less massive component filling the Roche lobe were named FO Vir type. As Shaw states, the massive components look in both types as normal, or “a bit evolved” Main Sequence (MS) stars, whereas low mass components are oversized for their masses:  $\approx 1.2$  times in V1010 Oph type and 2–3 times in FO Vir type binaries. NCBs of V1010 Oph type show shortening of the orbital period and O’Connell effect in their light curves, interpreted as resulting from a hot spot situated on the trailing side of the less massive component. NCBs of FO Vir type never show O’Connell effect and period lengthening prevails in them. Yakut and Eggleton (2005) suggested the name SD1 and SD2 for NCBs of V1010 Oph and FO Vir type, respectively. We will use their designations in the following.

The ranges of component masses and orbital periods of NCBs overlap with the ranges of cool contact binaries (CB), which implies a relationship between these two kinds of binaries. The shapes of light curve differ, however, substantially: while equal, or almost equal depth minima are observed for CBs, the minima of NCBs are distinctly different. SD1 binaries have sometimes been identified with the broken contact phase of the Thermal Relaxation Oscillation (TRO) model of CBs described in detail below. The presence of such a phase is an important constituent of this model. Because, however, SD2 binaries do not fit to the TRO model, several authors suggested that they may represent another route to CB formation, resulting from mass transfer between the components with mass ratio reversed (as in case of Algols). The evolutionary model of CBs suggested by one of us (KS) assumes that all cool CBs are formed this way and their components are in thermal equilibrium with no need for TROs. The main purpose of the present investigation is to show that the observed properties of NCBs cannot be reconciled with the TRO model but they fit well into our model. Before we place NCBs into the evolutionary sequence leading to CB formation, we describe first the historical development of our understanding of origin, evolution and observational properties of cool CBs.

Eclipsing binaries of W UMa-type were discovered more than a century ago (Müller and Kempf 1903). The analysis of the light and velocity curves carried out then showed that the binaries were very unusual, with the components in contact and the uniform surface brightness. According to the present definition, a cool CB, or W UMa-type binary consists of two lower MS stars surrounded by a common envelope lying between the inner and outer Lagrangian zero velocity equipotential surfaces (called also the Roche lobes) and possessing almost identical mean surface brightnesses (Mochnecki 1981). The more massive, primary component is a MS star, lying often close to Zero Age MS (ZAMS), and the secondary component is oversized, compared to its expected ZAMS size. The accepted upper limit is 1 d for the orbital period and  $2.5\text{--}3 M_{\odot}$  for the total mass of a cool CB although some authors prefer somewhat higher values. The observed lower limits for the orbital period and total mass are about 0.2 d and  $1.1 M_{\odot}$ , respectively.

Based on early observations, several papers explaining the observed properties of CBs were published in 50-ties and 60-ties of the past century but no satisfactory model was obtained. Since then, an enormous amount of new data have been collected on these stars, yet their structure and evolutionary status still remain obscured.

Following the original suggestion by Jeans (1928) it was believed for a long time that CBs are formed by fission of the protostellar core. An apparently satisfactory agreement of the observed basic parameters of W UMa type stars with predictions resulting from the fission mechanism was obtained by Roxburgh (1966). This strengthened the conviction that CBs are born as contact systems with a uniform chemical composition. Yet, a fundamental difficulty in explaining their structure remained. As Kuiper (1941) demonstrated, the ratio of the thermal equilibrium radii for two ZAMS stars is approximately directly proportional to the mass ratio whereas the geometry of the inner critical Roche lobes requires that it scales as, approximately, a square root of mass ratio. These two conditions can be fulfilled only when both components have identical masses. Two different zero age stars in thermal equilibrium do not fit into their Roche lobes. This is still true for the (coeval) binary components burning hydrogen in their cores as long as their masses are close to their initial values. The fact that we do observe contact binaries with different component masses, is known as the Kuiper paradox. Let us note in this place, however, that the Kuiper paradox does not apply to evolved binaries past mass exchange with mass ratio reversal (*e.g.*, Algols) because each component obeys a different mass-radius relation, depending on its evolutionary status. We will discuss this problem later.

A very convincing attempt to solve the Kuiper paradox was undertaken by Lucy (1968). To explain the existence of two unevolved stars in contact, he noted that the mass-radius relation for stars with proton-proton (pp) nuclear cycle has a different slope, compared to the same relation for stars with CNO cycle. As a result, the Kuiper paradox can be solved by assuming that the pp cycle dominates in one component and the CNO cycle dominates in the other one, and that convection zones of both stars share the same adiabatic constant. This situation requires a very efficient energy transport from primary to secondary component through the common part of the convective envelopes. However, detailed calculations showed that realistic models can only be produced within a narrow range of component masses, contrary to what was observed. A model giving more flexibility in selection of the component masses was suggested several years later also by Lucy (1976). He still came out from the two fundamental assumptions: first, that CBs are formed at ZAMS and second, that the specific entropy is identical in the convection zones of both components. To explain the Kuiper paradox for a broad range of component masses he abandoned the assumption of thermal equilibrium for each component separately. A similar model was concurrently developed by Flannery (1976). The model is known as TRO model. Both components are supposed to oscillate about

the equilibrium state, with the whole binary remaining in the global thermal equilibrium. The energy is transported from primary to secondary *via* a turbulent convection which results in equal entropies of both convective envelopes, hence equal surface brightnesses. Lucy did not consider details of the energy transfer; he simply assumed that on reaching contact between the components, entropies of both convection zones equalize instantaneously. Later, TRO model was also applied to initially detached binaries in which a primary reaches its Roche lobe after some time spent on MS (*e.g.*, Webbink 1976, 1977ab, Sarna and Fedorova 1989, Yakut and Eggleton 2005). Detailed modeling of such binaries shows that, following the Roche lobe overflow (RLOF), the primary transfers about  $0.1 M_{\odot}$  to the secondary which expands and fills its Roche lobe, forming a CB. Then, the TRO paradigm applies with a direction of secular mass transfer reversed, *i.e.*, from the secondary to primary, until both components merge.

TRO model explains two basic observational facts about W UMa-type stars: the geometry of the binary where the primary component is an ordinary MS star and the secondary is also a MS star but swollen to the size of its Roche lobe by energy transfer, and equal apparent effective temperatures of both components resulting in a characteristic light curve with two equal minima. It also gives an additional observational prediction, that a binary oscillates between two states: contact – when both stars fill their respective Roche lobes and mass flows from the secondary to the primary, and semidetached – when the primary still fills its Roche lobe but the secondary detaches from its lobe and mass flows from the primary to the secondary (Lucy 1976). Time scales of both states should be comparable to one another. Additional effects, like stellar evolution and/or angular momentum loss (AML) can influence the ratio of both time scales reducing the duration of the semidetached state (Robertson and Eggleton 1977, Rahunen 1983, Yakut and Eggleton 2005, Li, Han and Zhang 2005). In the most extreme case a binary can remain in contact all the time if AML rate is high enough but then its lifetime as a CB must be as short as  $\approx 10^8$  y as noted by Webbink (2003). Such binaries would be rare in space, contrary to observations.

As more and more theoretical and observational data on CBs are accumulated, the TRO model encounters increasing difficulties. In particular, its both basic assumptions, *i.e.*, zero age of CBs and identical specific entropy in the convection zones of both components, are questionable and its basic prediction of the broken contact phase seems to be at odds with observations.

Fission of a contracting protostellar core has been abandoned many years ago and it is no longer considered a feasible mechanism for binary formation. Numerical simulations of the bar-like instability developing in a rapidly rotating liquid body showed that it never results in a fission if the liquid is compressible (which a star is). Instead, a spiral arm structure develops resulting in a disk, or ring containing the excess of angular momentum (AM) and a stable core (Bonnell 2001). At present, the early fragmentation of a protostellar cloud resulting in two proto-

stellar cores orbiting each other is considered to be a dominating mechanism for binary formation (Machida *et al.* 2008 and references herein). In effect, the orbit size of a freshly formed binary must be large enough to accommodate both pre-MS components. In particular, the low period limit for the isolated progenitors of W UMa-type binaries with total masses between  $1.5 M_{\odot}$  and  $3 M_{\odot}$  is about 1.5–2.0 d. Recently, a mechanism for tightening of the originally wide, eccentric orbit by an interaction with a properly placed, distant third companion has been suggested. The interaction enforces the, so called, Kozai cycles which, together with the tidal friction (KCTF), make the period of the inner binary shorten. Detailed calculations showed that the inner period can be shortened down to a value of about 2–3 d only (Eggleton and Kiseleva-Eggleton 2006, Fabrycky and Tremaine 2007). The orbit is circularized at this value and the mechanism does not work any more. So, excluding very exceptional cases, we do not expect the KCTF mechanism to produce binaries with periods shorter than this limit. The typical time scale of the period shortening of a binary with the initial period of 10–20 d is of the order of  $10^6$ – $10^7$  y. We should then observe an excess of young binaries with periods of 2–3 d compared to the “canonical” Duquenoy and Mayor (1991) distribution. This is indeed observed among binaries in Hyades (Griffin 1985, Stępień 1995). Tokovinin *et al.* (2006) demonstrated, on the other hand, that almost all field binaries with periods shorter than 3 d are in fact triple systems, compared to only about one third of long period binaries possessing a tertiary companion. This suggests that the KCTF mechanism works indeed efficiently, producing large numbers of short period binaries compared to those formed as isolated systems.

The observations of pre-MS and young binaries are in a full agreement with the above expectations. HD155555 with  $P = 1.7$  d has the shortest known period among pre-MS binaries (although some authors suggest that it may already be on MS). All other binaries of T Tau type and members of young clusters with masses corresponding to CBs have periods longer than 2 d. We conclude that theoretical, as well as observational data show that detached binaries with initial periods of the order of 2–3 d are dominant progenitors of CBs. Substantially shorter periods of a fraction of a day may be encountered among young binaries only exceptionally, *e.g.*, as a result of a hard collision with another object in a dense stellar environment but they should be very rare (Bradstreet and Guinan 1994). Note that this conclusion restricts an acceptable range for initial orbital periods when modeling CBs. It is incorrect to adopt initial orbital periods shorter than, say, 1.5 d when considering a model of a typical contact binary, as some authors do (*e.g.*, Webbink 1976, 1977ab, Sarna and Fedorova 1989, and more recently Jiang *et al.* 2012).

Binaries with components possessing subphotospheric convection zones show hot coronae and magnetized winds carrying away mass and AM. This is a mechanism of magnetic braking (MB). The time scale for AML of a binary with a period of 2 d is approximately of the same order as the MS life time of the primary component (Stępień 2011a). We should expect then that a primary is close to, or even

beyond terminal age MS (TAMS) when it fills the Roche lobe. It can fill the Roche lobe at the earlier stages of MS life only if its mass is lower than about  $1.1 M_{\odot}$  and the initial period is shorter than 2 days (Śtepień and Gazeas 2012). In other words, RLOF occurs when a typical progenitor of CB is rather old, with an age comparable to the MS life time of its primary which under the circumstances has substantially, or even completely depleted hydrogen in the core. Such stars lie close to TAMS on the Hertzsprung-Russell diagram. As primaries retain their status within the framework of the TRO model until coalescence, we should not observe primaries of W UMa type stars close to ZAMS. If, however, CBs are past mass transfer with the mass ratio reversed, the present primary consists of a little evolved former secondary and matter coming from the outer layers of a former primary. These stars should lie close to ZAMS. The accurate data for a number of W UMa type stars show that most of their primaries lie at, or close to ZAMS (Yakut and Eggleton 2005, Śtepień 2006a, Siwak *et al.* 2010). Only the most massive primaries with masses higher than  $1.5 M_{\odot}$ , for which the evolutionary time scale shortens substantially as a result of increasing mass transferred slowly from secondaries, lie farther from ZAMS. Observations of W UMa-type stars in stellar clusters confirm their advanced age. They are absent in young and intermediate-age clusters while they appear abundantly in clusters with age exceeding 4–4.5 Gyr (Rucinski 1998, 2000). Kinematical analysis of field W UMa-type stars also shows that the binaries have an average age of several Gyr (Guinan and Bradstreet 1988, Bilir *et al.* 2005).

The second basic assumption of the TRO model deals with the energy transfer between the components. Lucy (1968, 1976) did not consider the mechanism for it. Instead, he assumed that the transport is very efficient so that entropies of both convection zones equalize immediately after a contact between the components is established. Detailed mechanisms of the energy transfer have been discussed by several authors. Applying different simplifying assumptions, some of them argued for turbulent, small-scale transport (*e.g.*, Moses 1976), while others considered large-scale circulations (Hazlehurst and Meyer-Hofmeister 1973, Webbink 1977c, Robertson 1980). However, no satisfactory result was obtained. As Yakut and Eggleton (2005) summarized: “... there is still remarkably little understanding of how the heat transport manages to be as efficient as it must be”. The problem of the energy transfer can be solved if the assumption of the equal entropy in both convection zones is dropped (see below).

In addition, the prediction of the TRO model that each CB should spend part of its life as a semi-detached system finds no observational support. Rucinski (1998) noted that CBs with distinctly different depths of minima (suggesting poor thermal contact) are quite rare. A recent photometric sky survey ASAS (Pojmański 2002) detected several thousand eclipsing binaries with periods shorter than 1 d in the solar neighborhood (Paczynski *et al.* 2006). Classification of the light curves resulted in a significant proportion of semi-detached systems. This would seemingly solve the problem and support TRO model, yet a closer look at the sample shows

that there is very few semi-detached binaries among stars with periods shorter than 0.45 d (Pilecki 2010) whereas an overwhelming majority of CBs has periods within this range (Rucinski 2007). On the other hand, semi-detached binaries are quite common among binaries with periods between 0.7 d and 1 d where few CBs are observed. Moreover, values of global parameters were obtained recently for several CBs and some NCBs using the high-precision photometric and spectroscopic observations, together with improved modeling procedure. The results show that the global parameters of NCBs are distinctly different from those of CBs as will be demonstrated later. So, the problem of the lack of binaries in a semi-detached phase of TRO model still remains.

To solve the problems with the existing model of cool CBs, an alternative model has been developed by one of us (Śtepień 2006ab, 2009, 2011a). The model assumes that CBs originate from young cool binaries with initial orbital periods close to 2 d. Both components rotate synchronously with the orbital period and have initial masses lower than  $1.3 M_{\odot}$ . Such stars show strong magnetic activity resulting in MB. Permanent AML makes the orbit tighten and the components approach each other until RLOF by the primary occurs. Rapid mass transfer follows, up to the equalization of the component masses. Mass transfer continues until both components regain thermal equilibrium. The NCB configuration is identified with the mass transfer phases when at least one component is out of thermal equilibrium. After regaining thermal equilibrium by both components, a contact, or Algol-type configuration is assumed. Depending on the amount of AM left in the system, mass transfer rate and AML rate, the Algol-type configuration may then evolve into contact or may widen the orbit and stay as semi-detached until the presently more massive component leaves MS and a common envelope develops. The evolution of several exemplary CBs was computed by Śtepień (2006a), Gazeas and Śtepień (2008) and Śtepień and Gazeas (2012) whereas a more systematic calculations of the cool binary evolution from the initial state till RLOF by the primary component in a number of systems with different initial masses and orbital periods was carried out by Śtepień (2011a). Energy flow by a large scale circulation between the components of a contact binary past rapid mass exchange was considered in detail by Śtepień (2009).

The present paper investigates the relatively short-lasting but crucial evolutionary phase of a cool close binary when the mass exchange occurs following RLOF and the binary assumes a NCB configuration. Section 2 considers in detail the process of mass transfer including, hitherto neglected, dynamics of the matter flowing from a donor to an accretor. The flow forms an equatorial bulge influencing the accretion rate when the accretor almost fills its Roche lobe. Its presence lengthens substantially the SD1 phase of the mass transfer. This explains the observed similar frequency of SD1 binaries, compared to SD2 binaries. In Section 3 observational data are analyzed. In particular, it is demonstrated that the basic parameters of SD1 and SD2 binaries are very similar to each other which favors the view that they cor-

respond to a single process of mass transfer. The parameters of NCBs are, on the other hand, distinctly different from the observed parameters of CBs which contradicts an assumption that SD1 binaries correspond to the broken contact phase of CBs. Section 3 contains a discussion of the results and Section 4 gives conclusions.

## 2. RLOF and the Mass Transfer

### 2.1. Early Phase of Mass Transfer

Following RLOF mass transfer begins. Matter flows from a donor filling the Roche lobe through the saddle point at the Lagrangian point L1 and falls onto the accretor lying deep inside its Roche lobe. Most of the papers considering the mass transfer were concentrated on the dynamics of the stream of matter leaving L1 and its interaction with the accretor. Dynamics of the surface layers of a donor did not attract much attention although it does influence the mass loss process.

The problem of a surface flow in a donor was considered in an approximate way by Lubow and Shu (1975). They demonstrated the existence of horizontal pressure gradients with the highest pressure at the poles and the lowest at L1. The pressure pattern results in horizontal currents similar to geostrophic circulation driven by the Coriolis force and known from the Earth atmosphere. The authors called this circulation “astrostrophic”. The frictional drag exerted on the flow by lower layers results in a slow drift of matter toward the equator. Once the matter reaches the equator it can flow to the L1 region. This flow pattern is in contradiction to the model considered by Webbink (1977c) who assumed that matter leaving the donor comes from the vertical expansion of gas at the immediate vicinity of L1. The velocity of this gas is low enough for the Coriolis force to be negligible. Recent full 3D hydrodynamic computations of the mass flow in the surface layers of a donor carried out by Oka *et al.* (2002) contradicted the flow pattern envisaged by Webbink (1977c) and confirmed the results of Lubow and Shu (1975). Gas elements starting at a high astrographic latitude circle the pole where high pressure occurs, drift to the equatorial region, run around the neck between the stars and flow through L1. The stream leaves the donor with velocity equal to the sound velocity and is deflected by the Coriolis force from the line joining the centers of both stars. The same flow pattern over the donor’s surface, dominated by the Coriolis force, was obtained in numerical simulations by Fujiwara *et al.* (2001).

The stream deflected by the Coriolis force misses the stellar surface and forms a disk when the accretor radius is small enough. Otherwise the stream strikes the surface at an inclined angle. The radial component of velocity is dissipated and heats the in-falling matter whereas the tangential component makes the matter flow around the star. In effect, a hot spot appears at the impact region and an equatorial bulge encircling the accretor is formed. The bulge is kept together by the Coriolis



force. The turbulent friction at the bottom of the equatorial stream produces the Ekman flow to the poles so that the in-falling matter ultimately covers the whole stellar surface whereas its excess AM (relative to the accretor) is transferred to stellar spin.

Apart from dynamical effects, the reaction of both stars to the mass transfer has been investigated by several authors for a range of component masses. Mass transfer in low mass binaries, *i.e.*, possible progenitors of W UMa-type stars, was considered, among others, by Yungelson (1973), Webbink (1976, 1977ab), Nakamura (1985), Sarna and Fedorova (1989), Eggleton and Kiseleva-Eggleton (2002), Yakut and Eggleton (2005), and recently by Ge *et al.* (2010). The reaction of a donor depends on its mass; stars with masses higher than  $0.9 M_{\odot}$  (assumed to be a low mass limit for primaries in progenitors of the field W UMa-type variables, see Stępień 2006b) shrink when losing mass (Ge *et al.* 2010) so, if only the mass ratio is not too low, the mass transfer proceeds on a thermal time scale of the donor. For the mass ratio  $q = M_{\text{acc}}/M_{\text{d}} < 1$ , where  $M_{\text{acc}}$  and  $M_{\text{d}}$  are the accretor and donor mass, respectively, the accretor reacts with swelling because its thermal time scale is longer than that of the donor, so it can not accommodate a (too high) mass transfer rate. Note that, due to a presence of the equatorial bulge, the accretor surface deviates from an equipotential surface because the height of the bulge amounts to a few percent of the stellar radius (see below). Nevertheless, the presence of the bulge does not influence significantly the global stellar parameters of the accretor as long as its top is within the inner Roche lobe. The situation changes, however, when the expanding accretor approaches the critical Roche surface.

## 2.2. Mass Transfer in the Near Contact Phase

Let us first qualitatively discuss the physical processes taking place when the expanding accretor almost fills its inner Roche lobe.

The height of the equatorial bulge amounts to a few percent of the stellar radius above the surface, so when the radius of the accretor approaches the size of the Roche lobe by this amount, the top of the bulge begins to stick out beyond the inner critical surface. In effect, instead of being accreted, part of the matter flows above the Roche lobe and returns to the donor. The accretion rate decreases until the accretor can accommodate the accreted matter and stops the expansion. The star stays inflated with the radius slightly smaller than the Roche lobe size and its mass increases at a rate dictated by its own thermal time scale. If the Roche lobe descends onto the stellar surface due to a period shortening, all matter from the stream returns to the donor and the binary behaves as a contact system described by Stępień (2009), until the accretor shrinks and dives beneath the Roche lobe. As a result, the accretor is surrounded by a thick circumstellar flow of which only a small fraction adds to its mass. As time goes on, the mass ratio approaches unity and so does the ratio of the thermal time scales of both components. The binary temporarily assumes a contact configuration overfilling the inner Roche lobe and, possibly,

even the outer lobe. In the latter case, a fraction of the binary mass (and AM) may be lost from the system. In the lack of accurate models of this process, we neglect these losses to avoid introducing arbitrary parameters describing them. When the thermal time scale of the donor becomes longer than that of the accretor, the latter star can accommodate the, now decreasing, flux of matter and its radius shrinks to the thermal equilibrium value. Also the donor returns to thermal equilibrium.

We will follow Stępień (2009) in a quantitative description of the flow dynamics in the bulge close to the Roche lobe.

The fluid flow in a frame of reference rotating with a binary is described by the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

and momentum equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} - \nabla \phi - 2\boldsymbol{\Omega} \times \mathbf{v}. \quad (2)$$

Here  $\rho, p$  and  $\mathbf{v}$  denote gas density, pressure and velocity,  $\phi$  is the gravitational (plus centrifugal) potential,  $\nu$  is kinematic viscosity coefficient and  $\boldsymbol{\Omega}$  is angular velocity. A full discussion of the flow would also require the energy equation but we can skip it when considering only dynamics of the transferred matter.

As was demonstrated by Stępień (2009), the inertial term and the viscous terms in Eq. (2) can be neglected in contact binaries. Here, we apply the same approximations. The resulting motion equation for a stationary flow reads

$$\frac{1}{\rho} \nabla p = 2\boldsymbol{\Omega} \times \mathbf{v} + \nabla \phi. \quad (3)$$

We further ignore the region close to the neck between the stars where the flow adjusts its size to the equilibrium condition and we concentrate on the equatorial bulge where  $\mathbf{v}$  has only one, non-vanishing azimuthal component  $v$ . The equation describes the flow confined by the Coriolis force with the width depending primarily on the flow velocity.

Let us introduce a spherical system of coordinates with the origin at the accretor center  $(r, \vartheta, \phi)$  and let us neglect the deviation of equipotential surfaces from spheres (Fig. 1). Because the stream is symmetric relative to the equator, we consider only a hemisphere  $0 \leq \vartheta \leq \pi/2$ . Far from the neck matter flows along the equator of the accretor in a belt contained between equator and the boundary given by angle  $\vartheta_f(r)$  (Stępień 2009)

$$\sin \vartheta_f = 1 - \frac{p_0}{2\rho_0 v v_{\text{eq}}} = 1 - \frac{c_s^2}{2\gamma v v_{\text{eq}}}, \quad (4)$$

where  $p_0$  and  $\rho_0$  are the pressure and density inside the bulge at the equator (they vary with the height above the accretor surface),  $v_{\text{eq}} = \Omega R_{\text{acc}}$  is the equatorial

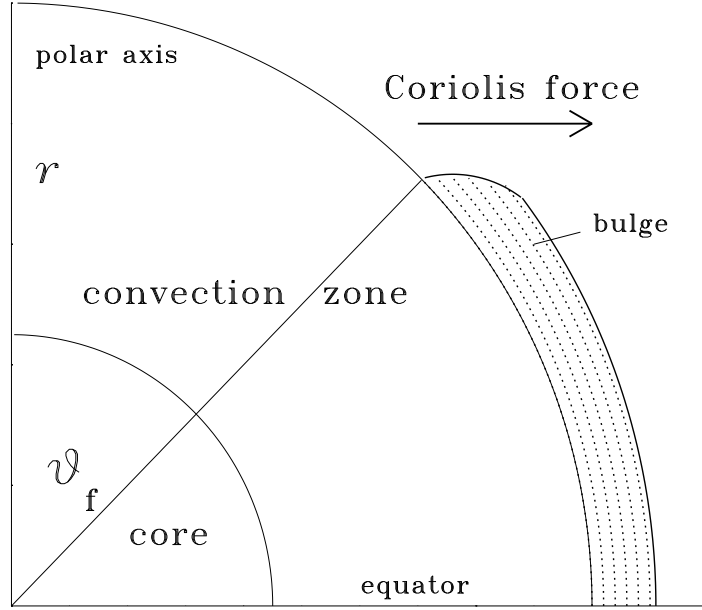


Fig. 1. The reference frame and geometry of the flow in the meridional plane of the accretor. The stream flows away, perpendicularly to the plane of the paper, in the  $\phi$ -direction. Scale for the flow thickness has been exaggerated.

velocity of the accretor with the radius  $R_{\text{acc}}$  and  $c_s = (\gamma p_0 / \rho_0)^{1/2}$  is the sound velocity inside the bulge.

In an inviscid case the matter streaming from the donor is collected in the bulge bound by the Coriolis force. As the matter accumulates, the increasing gas pressure, density and temperature at the bottom of the bulge results in its broadening, see Eq. (4), until  $c_s^2 = 2\gamma v v_{\text{eq}}$  when the bulge reaches the poles. The non-vanishing viscosity will accelerate spreading out the bulge matter by the Ekman drift. Molecular viscosity is negligible but because the surface layers of the accretor, as well as the bulge, are in a turbulent state, the turbulent viscosity will influence the flow.

The convective viscosity  $\nu = l_{\text{conv}} v_{\text{conv}} / 3$ , where  $l_{\text{conv}}$  and  $v_{\text{conv}}$  are the mixing length and convective velocity near the bottom of the stream. The estimate of the viscosity term in Eq. (2) gives  $\nu v / \Delta x^2 \approx l_{\text{conv}} v_{\text{conv}} v / 3 R_{\text{acc}}^2$  so the ratio of the viscous term to the Coriolis term,  $k_{\text{drift}}$ , is given by

$$k_{\text{drift}} = \frac{l_{\text{conv}} v_{\text{conv}}}{6 R_{\text{acc}} v_{\text{eq}}}. \quad (5)$$

Assuming that  $l_{\text{conv}}$  is of the order of pressure scale height near the bottom of the bulge we obtain from the model of the convective zone  $H_p \approx 10^9$  cm and  $v_{\text{conv}} \approx 2 \times 10^4$  cm/s (Stepień 2009). Adopting for the orbital binary period  $P_{\text{orb}} = 0.6$  d as a typical value for NCBs (see Section 3) and  $R_{\text{acc}} \approx R_{\odot}$  we finally obtain

$k_{\text{drift}} = 5.7 \times 10^{-6}$ . This value shows that the viscous term can indeed be neglected when considering the azimuthal motion but the drift velocity perpendicular to the flow motion is of the order of  $k_{\text{drift}}v$ . The resulting drift time to reach a pole is  $t_{\text{drift}} = R_{\text{acc}}/k_{\text{drift}}v \approx 150$  y if we adopt a value of 50 km/s for  $v$ , equal to the sound velocity in a layer close to the inner Roche lobe of the donor (Oka *et al.* 2002). The result indicates that the bulge will grow for the first 150 y of the mass transfer and then a stationary state will be reached with matter drifting away at the same rate as flowing in from the donor. With a typical mass transfer rate of  $1-2 \times 10^{-7} M_{\odot}/\text{y}$  about  $1-2 \times 10^{-5} M_{\odot}$  is deposited in the bulge and its total width is  $\pm 30^\circ$  from the equator, as it can be seen from Eq. (4). Assuming an adiabatic structure inside the bulge, its height resulting from the model of the convection zone is equal to  $\approx 3\%$  of the accretor's radius. If a back pressure along the equator is significant, the flow velocity in the bulge may be lower than the sound speed. A decrease of the flow velocity *e.g.*, by a factor of 2 gives a value of 300 y for the time scale of the Ekman drift to the poles and  $\pm 45^\circ$  for the width of the bulge. The change of the bulge height is insignificant.

To summarize, the presence of the bulge makes an equatorial radius of the accretor vary between about 97% and 100% of the inner Roche radius during the phase of the mass transfer before the mass ratio reversal. As long as the bulge top is inside the Roche lobe, its temperature (besides hot spot) should be close to the surface temperature of the accretor because the thermal time scale of the accreted matter is much shorter than the drift time. But when the bulge top sticks considerably above the Roche lobe and the binary is in NCB phase, most of the matter flowing in it is replaced by a fresh, hot matter from the donor after just one revolution around the accretor equator. As a result, the temperature of the bulge becomes close to the temperature of the donor surface layers (Śtepień 2009) whereas the rest of the accretor surface is cooler. The mean surface temperature of the accretor, averaged over the bulge and the polar regions, is expected to rise relative to the unperturbed value although it still remains lower than the donor's temperature. The deviation of the accretor shape from the equipotential surface and the non-uniform temperature distribution make difficult, if not impossible, unique determination of NCB geometrical parameters, based only on the analysis of the light curve.

Accepting the TRO paradigm, some authors stopped evolutionary calculations of the mass transfer after the accretor filled its Roche lobe and a contact configuration was formed (*e.g.*, Sarna and Fedorova 1989, Yakut and Eggleton 2005). They assumed a rapid equalization of entropies in the convection zones of both components resulting in thermal oscillations, with a reversed secular mass transfer (*i.e.*, from the accretor to the donor). With an energy transfer due to an equatorial flow, as described by Śtepień (2009), this equalization does not occur and mass transfer can proceed until mass ratio reversal, just as in case of massive binaries forming classical Algols. Detailed models of mass transfer ignoring the assumption

of the forced equality of entropies confirm this scenario (Webbink 1976, Nakamura 1985). In particular, let us discuss details of the model calculations carried out by Webbink (1976). He considers a binary with component masses equal to  $1.5 M_{\odot}$  and  $0.75 M_{\odot}$  and the initial orbital period of 0.74 d. The evolution was strictly conservative, *i.e.*, without MB. The primary fills its Roche lobe (and becomes a mass donor) after about 1.53 Gyr (60% of its MS life) when the core hydrogen content has been reduced to 0.168. Allowing for MB, binaries with initial periods of about 1.5 d would reach RLOF after a similar evolutionary advancement of the primary (Śtepień 2011a) so, apart from a different evolutionary history prior to RLOF, the results of calculations by Webbink can be applied to some of our models.

Following RLOF, the donor transfers mass at a (maximum) rate  $\dot{M}_d \approx M_d/t_{\text{th}}$  (Paczynski 1971) which for the  $1.5 M_{\odot}$  star gives  $\dot{M}_d \approx 2.5 \times 10^{-7} M_{\odot}/\text{y}$ . We adopt  $t_{\text{th}} \approx 6 \times 10^6$  y for a  $1.5 M_{\odot}$  star approaching TAMS and  $1.5 \times 10^8$  y for an  $0.85 M_{\odot}$  star at ZAMS ( $0.85$  results from an initial  $0.75 M_{\odot}$  mass plus  $0.1 M_{\odot}$  accreted in the first phase of mass transfer). Transferring one tenth of the solar mass takes less than one million years. More accurate model computations give about 2 mln years (Webbink 1976). After accreting that amount of mass the accretor fills in its Roche lobe. Assuming that the accretor still accepts the transferred mass, additional  $0.25 M_{\odot}$  is transferred within the next 5 mln years resulting in approximate equality of component masses, which ends the second mass transfer phase. From now on the mass transfer rate is low enough that the accretor can accommodate it in thermal equilibrium. It detaches from the Roche lobe and the binary assumes an SD2 configuration. This phase takes again about 5 mln years. The next phase of mass transfer follows with the mass transfer rate still relatively high and the donor returning to thermal equilibrium. Because of the presently low mass of donor, it takes about  $7 \times 10^7$  y. Both components reach thermal equilibrium with masses  $0.96 M_{\odot}$  and  $1.29 M_{\odot}$  for the donor and accretor, respectively and with the orbital period of about 0.55 d (see Fig. 12 and Table 2 in Webbink 1976). Mass transfer continues at a highly reduced rate, determined only by the evolution of the donor because the process is strictly conservative. Recent models include MB which makes the orbit contract so that the mass transfer rate in this phase results from a balance between the evolutionary expansion of the donor and MB. Depending on the relative importance of these two processes, the orbital period will shorten making the binary evolve to contact configuration or the period will lengthen, making it evolve to the extreme mass ratio Algol. As we argued above, the picture of the rapid mass transfer described by Webbink (1976) is generally correct, except that the time scale of the second mass transfer phase, prior to mass ratio reversal, is longer by an approximate ratio of the thermal time scales of accretor and donor, *i.e.*, by about 1–1.5 orders of magnitude. This increases its duration to about  $5 \times 10^7$  y in the considered case. The binary is of SD1 type at this phase. Note that this duration is close to  $7 \times 10^7$  y spent in SD2 configuration (see above).

### 3. Comparison with Observations

#### 3.1. Global Parameter Distributions

According to the evolutionary scenario presented above, the NCB phase describes the transition between detached and contact or Algol-type binary. SD1 configuration precedes SD2 configuration and each of them takes only a small fraction of a total cool binary life. Each binary goes only once through both NCB phases. This is in contrast to the TRO model which assumes that most of SD1 stars are in a broken contact phase and a binary oscillates many times between contact and SD1 phase (Lucy 1976, Flannery 1976, Robertson and Eggleton 1977). Predictions of both evolutionary models can be verified observationally.

Cool close binaries lose mass and AM *via* MB during their evolution, so these two parameters decrease monotonically along an evolutionary sequence. Binaries in earlier evolutionary phases are expected to have, on average, higher values of the total mass and orbital AM, compared to similar binaries in later evolutionary phases. Binaries in the same, or almost the same evolutionary phase should show the same properties regarding the orbital period, total mass and AM. In addition, the observed frequency of stars in a given phase should be proportional to the duration of this phase.

In Table 1 we collected available data from literature on NCBs for which masses have been determined. Only binaries with a total mass lower than  $3 M_{\odot}$  and periods shorter than 1 d were included. The  $3 M_{\odot}$  limit was set to avoid early type binaries which have different origin, do not possess subphotospheric convection zones and are not progenitors of W UMa-type stars. Note that this limit is somewhat higher than  $2.6 M_{\odot}$  adopted in our earlier model calculations. Consecutive columns in Table 1 give star name, period, component masses, orbital AM, type (SD1 or SD2) and references. These data will be compared with the observational data on W UMa-type stars given by Gazeas and Stępień (2008). Two contact binaries with total masses higher than  $3 M_{\odot}$  are omitted and the newer data for AW UMa are accepted (Pribulla and Rucinski 2008). There are 22 SD1 stars and 27 SD2 stars in Table 1 and they will be compared with 110 W UMa-type stars.

There exist considerable differences in quality of both sets of data. All W UMa-type stars were observed spectroscopically and have good light curves. A simultaneous solution of radial velocity and light curves results in accurate values of stellar parameters. In case of NCB stars, very few have radial velocity curves in addition to a light curve, resulting in accurate values of their parameters (see *e.g.*, Siwak *et al.* 2010). In most cases only a light curve of a moderate quality is available which results in uncertain values of stellar parameters. Sometimes even for stars with good radial velocity curves divergent data are obtained. RZ Dra is an example. Spectroscopic mass ratio  $q = 0.4$  was found by Rucinski and Lu (2000). Yet Yang *et al.* (2010) found  $1.91 M_{\odot}$  and  $0.72 M_{\odot}$  for the component masses ( $q = 0.38$ ) whereas Erdem *et al.* (2011) give  $1.63 M_{\odot}$  and  $0.70 M_{\odot}$  ( $q = 0.43$ ). We decided

Table 1

Data on NCBs

Name	Period [d]	Masses [ $M_{\odot}$ ]	$H_{\text{orb}} \cdot 10^{51}$ [cgs]	Type	Lit.	Name	Period [d]	Masses [ $M_{\odot}$ ]	$H_{\text{orb}} \cdot 10^{51}$ [cgs]	Type	Lit.
BL And	0.7224	1.80+0.70	10.316	SD1	Z06	DZ Cas	0.7849	1.89+0.17	2.882	SD2	Y12b
CN And	0.4628	1.43+0.55	5.995	SD1?	S10	IV Cas	0.9985	1.98+0.81	14.077	SD2	K10
NP Aqr	0.8070	1.65+0.99	13.660	SD1	I10	EG Cep	0.5446	1.65+0.77	9.554	SD2	Z09a
V609 Aql	0.7966	1.05+0.72	7.193	SD1	T08	YY Cet	0.7905	1.78+0.92	13.479	SD2	W12
DO Cas	0.6847	2.13+0.65	10.735	SD1	S10	TW CrB	0.5889	1.20+0.87	8.518	SD2?	F12
V473 Cas	0.4155	1.00+0.48	3.911	SD1?	Z09b	W Crv	0.3881	1.00+0.68	5.182	SD2	R00
V747 Cen	0.5371	1.54+0.49	6.017	SD1?	S10	RV Crv	0.7473	1.64+0.45	6.473	SD2	M86a
VV Cet	0.5224	1.04+0.30	2.822	SD1?	S10	V836 Cyg	0.6534	2.20+0.78	12.807	SD2	Y05
WZ Cyg	0.5845	1.59+1.00	11.971	SD1?	S10	RZ Dra	0.5509	1.77+0.71	9.226	SD2	E11
BV Eri	0.5077	1.56+0.43	5.283	SD1?	S10	AX Dra	0.5681	1.46+0.92	10.342	SD2?	K04
TT Her	0.9121	1.56+0.68	9.741	SD1	M89	FG Gem	0.8191	1.75+0.71	10.681	SD2	LN11
FS Lup	0.3814	1.30+0.61	5.740	SD1?	S10	GW Gem	0.6594	1.74+0.80	11.013	SD2	L09
FT Lup	0.4701	1.87+0.82	10.628	SD1?	S10	SZ Her	0.8181	1.58+0.75	10.373	SD2	L12
SW Lyn	0.6441	1.72+0.90	11.998	Det?	KR03	AV Hya	0.6829	2.38+0.62	11.155	SD2	Y12a
UU Lyn	0.4685	2.10+0.74	10.578	SD1?	Z07	DD Mon	0.5680	1.29+0.87	8.925	SD2	P09
V361 Lyr	0.3096	1.26+0.87	7.149	SD1	H97	DI Peg	0.7118	1.18+0.70	7.402	SD2	LU92
V1010 Oph	0.6614	1.89+0.89	12.914	SD1	S10	HW Per	0.6348	1.66+1.10	13.836	SD2	S98
BO Peg	0.5804	1.90+1.00	13.776	SD1	Y86	AG Phe	0.7553	1.42+0.22	2.997	SD2	C96
RT Scl	0.5116	1.63+0.71	8.635	SD1	H86	ROSAT (1)	0.3881	0.92+0.82	5.633	SD2	LI09
GR Tau	0.4299	1.45+0.32	3.590	SD1	G04	V1241 Tau	0.8230	1.60+0.87	11.950	SD2	Y12b
V1374 Tau	0.2508	0.67+0.32	1.680	SD1?	A11	BF Vel	0.7040	1.98+0.84	12.960	SD2	M09
BS Vul	0.4760	1.52+0.52	6.050	SD1	Z12	BF Vir	0.6406	1.23+0.41	4.569	SD2	R81
CX Aqr	0.5560	1.19+0.64	6.363	SD2	M86b	FO Vir	0.7760	2.00+0.26	4.522	SD2?	R05
ZZ Aur	0.6012	1.62+0.76	9.643	SD2	O06	AW Vul	0.8065	1.60+0.87	11.901	SD2	LN11
HL Aur	0.6225	1.50+1.10	12.686	SD2?	Z97						

ROSAT(1)=1RXS J201607.0251645 Lit.: A11=Austin *et al.* 2011, C96=Cerruti 1996, E11=Erdem *et al.* 2011, F12=Faillace *et al.* 2012, G04=Gu *et al.* 2004, H86=Hilditch and King 1986, H97=Hilditch *et al.* 1997, I10=Ibanoglu *et al.* 2010, K04=Kim *et al.* 2004, K10=Kim *et al.* 2010, KR03=Kreiner *et al.* 2003, L09=Lee *et al.* 2009, L12=Lee *et al.* 2012, LI09=Li *et al.* 2009, LN11=Liakos *et al.* 2011, LU92=Lu 1992, M09=Manimanis *et al.* 2009, M86a=McFarlane *et al.* 1986a, M86b=McFarlane *et al.* 1986b, M89=Milano *et al.* 1989, O06=Oh *et al.* 2006, P09=Pribulla *et al.* 2009, R00=Rucinski and Lu 2000, R05=Rucinski *et al.* 2005, R81=Russo and Sollazzo 1981, S10=Siwak *et al.* 2010, S98=Samec *et al.* 1998, T08=Turner *et al.* 2008, W12=Williamon and Sowell 2012, Y05=Yakut *et al.* 2005, Y12a=Yang *et al.* 2012a Y12b=Yang *et al.* 2012b Y86=Yamasaki and Okazaki 1986, Z06=Zhu and Qian 2006, Z07=Zhu *et al.* 2007, Z09a=Zhu *et al.* 2009a, Z09b=Zhu *et al.* 2009b, Z12=Zhu *et al.* 2012, Z97=Zhang *et al.* 1997

to list arithmetic means of both results in Table 1. It is well known that the mass ratio is poorly constrained when only light curve is available, particularly for low orbit inclinations. In addition, the commonly used Wilson-Devinney program for solving light curves gives often ambiguous solutions with marginally different quality of fit between SD1 and SD2 configuration (Qian *et al.* 2006) or even among NCB, contact and detached configuration (Odell *et al.* 2009). The ambiguity results in multiple classification of some stars, depending on subtle details of the analyzed light curve or, possibly, on the intention of the author(s) who may *e.g.*, prefer the SD1 solution for a binary with a decreasing period. Examples of divergent classifications are given by Siwak *et al.* (2010). All three data samples suffer from an observational bias toward bright objects. Nevertheless, we believe that the samples are sufficiently representative to draw meaningful conclusions about their statistical properties.

We first compare the observed frequencies of binaries in different evolutionary phases with the expectations resulting from our model. The analysis of V361 Lyr by Hilditch *et al.* (1997) indicates that the binary is in the initial phase of mass transfer when the accretor has not yet expanded in reaction to the matter flowing onto it. The light curve shows a prominent hump caused by a hot spot on the accretor surface. The star should approach its Roche lobe in about 2 mln years as the calculations by Sarna and Fedorova (1989) show. Similar time scale was obtained by Webbink (1976) and Nakamura (1985). Recently, another faint star, 2MASS J05280799+7256056, was detected showing a light curve with a similar hump (Virnina *et al.* 2011). According to the “classical” model by Webbink (1976) and Nakamura (1985) the next phase of the mass transfer, until the component mass equalization, takes only 2–3 times longer than the initial phase. We suggest that it takes about 1–1.5 orders of magnitude longer. The observed number of the V361 Lyr-type variables (one, or possibly two) can be compared to the number of the other SD1 variables listed in Table 1, which is 21. Their ratio is not in contradiction with our predictions and suggests a much longer duration of the total SD1 phase than that of V361 Lyr-like.

A similar comparison can be made for the duration of SD1 vs. SD2 phase. Our scenario predicts that each of them takes approximately the same time interval. This is in a fair agreement with the numbers of binaries in each configuration, listed in Table 1: 22 of SD1 vs. 27 of SD2 binaries.

Our model assumes that SD2 phase follows SD1 phase and that they both last rather shortly, compared to the total evolutionary life time of a binary. For a conservative mass exchange, global parameters of a binary should not change significantly between these two phases. We can check this by comparing both samples of NCBs. The average total mass of SD1 binaries listed in Table 1 is equal to  $2.20 \pm 0.11$  and of SD2 binaries to  $2.32 \pm 0.08$ . Both numbers differ insignificantly. The same conclusion is drawn regarding the mean values of the orbital angular momentum. Here we have  $8.20 \pm 0.76$  and  $9.23 \pm 0.64$  ( $\times 10^{51}$  in cgs units), for SD1 and SD2



binaries, respectively. Only the average orbital periods of both groups of NCBs differ significantly:  $0.55 \pm 0.03$  d vs.  $0.67 \pm 0.03$  d, for SD1 and SD2, respectively. However, orbital period is not a monotonic function of time. As we explained above, some of SD1 binaries evolve quickly to a contact configuration by shortening their periods (or, possibly, by mass and AML in a non-conservative case of mass transfer) whereas the others evolve to an Algol configuration by lengthening their periods (Stępień 2011b). The former binaries fall out because they are classified as CBs. On the other hand, the boundary between stars classified as SD2-type binaries and short-period Algols is not well defined. As a result, there are SD2 binaries listed in Table 1 with a primary filling only about 80% of its Roche lobe (SZ Her or V1241 Tau) and longer than average periods. Had we moved them to a category of usual Algols, we would have obtained a lower value for an average period of SD2 binaries. So, we do not consider the difference in average periods of SD1 and SD2 binaries as meaningful.

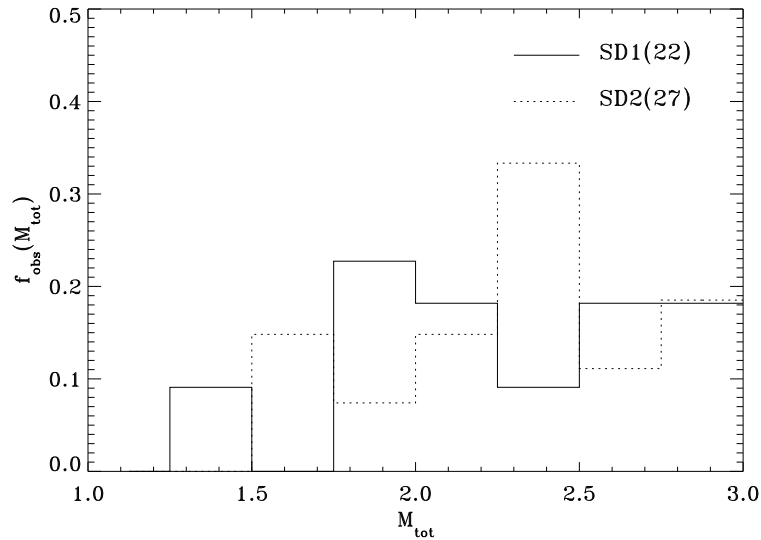


Fig. 2. Mass distributions of NCBs from Table 1. Both distributions are normalized to unity. Solid and dotted lines describe data for SD1 and SD2 binaries, respectively.

With sufficiently numerous samples it is possible to compare not only mean values but also the distributions of the discussed parameters. Figs. 2, 3 and 4 show the mass, AM and period distributions of both kinds of NCBs. It is easy to see that the respective distributions look very similar. The statistical analysis supports this impression. The  $\chi^2$  – test applied to all three pairs of the distributions gives values of the reduced  $\chi^2$  equal to 1.71, 0.64 and 1.65 for mass, AM and period distributions, respectively. Null hypothesis that both samples were drawn from different distributions can be rejected at the 5% level, although the probability of different distributions in case of the total mass and period is slightly higher than in case of AM.

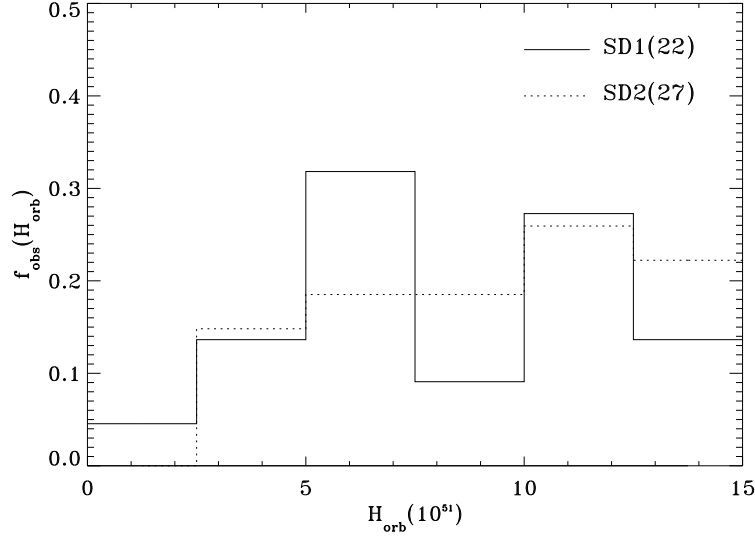


Fig. 3. Orbital angular momentum distributions of NCBs from Table 1.

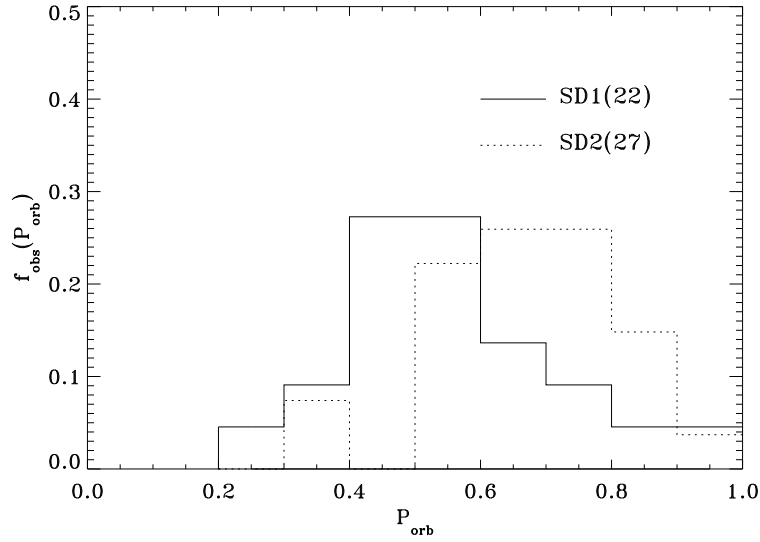


Fig. 4. Orbital period distributions of NCBs from Table 1.

We conclude that no significant differences exist between both samples of NCBs. So, we merge them together and compare with CBs from Gazeas and Stepień (2008), as described above. A mean value of the total mass of CBs is equal to  $1.81 \pm 0.04 M_{\odot}$  which is substantially less than  $2.27 \pm 0.06 M_{\odot}$  – a mean value for all NCBs. A mean value of the orbital AM of CBs is equal to  $4.68 \pm 0.25$  which can be compared to  $8.73 \pm 0.50$  – a mean value for NCBs. Again, the difference is substantial. Finally, mean values of the period are  $0.42 \pm 0.01$  d and  $0.61 \pm 0.02$  d,

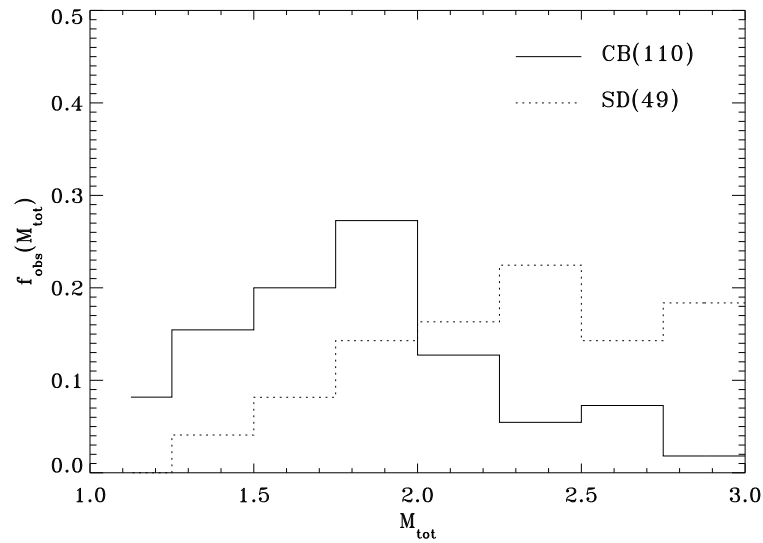


Fig. 5. Mass distributions of all NCBs from Table 1 and CBs from the paper by Gazeas and Stępień (2008), see text. Both distributions are normalized to unity. Solid and dotted lines describe data for CBs and NCBs, respectively.

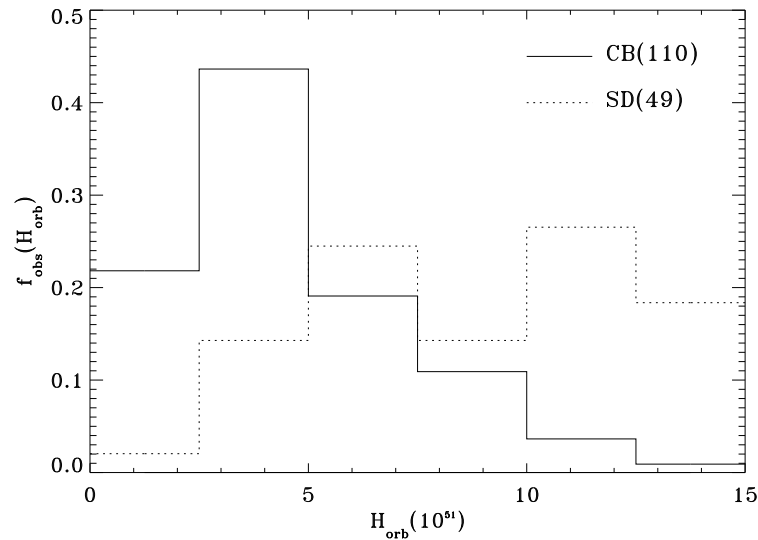


Fig. 6. Orbital angular momentum distributions of all NCBs from Table 1 and CBs from the paper by Gazeas and Stępień (2008).

for CBs and NCBs, respectively. They also are substantially different. The TRO model predicts period variations in the course of an oscillation but the expected amplitude does not exceed 10% (Kähler 2002). The observed difference is several times larger. Figs. 5, 6 and 7 show the distributions of the respective parameters for both groups of binaries. It is immediately visible that the distributions are different.

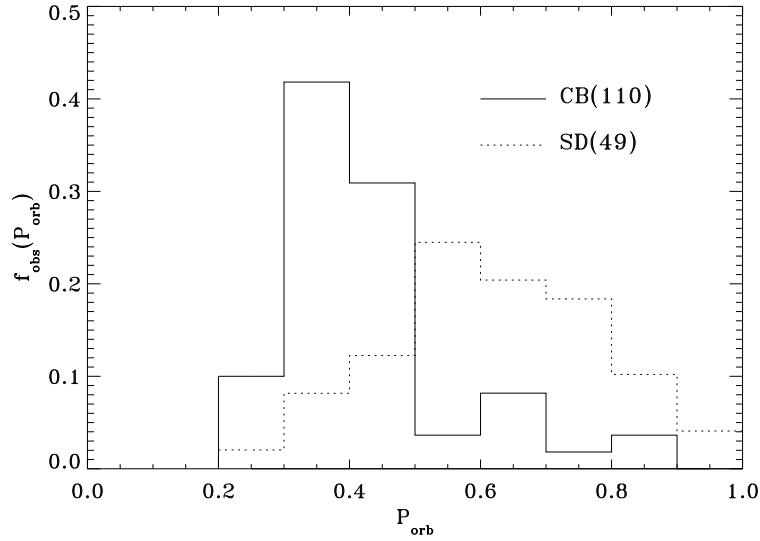


Fig. 7. Orbital period distributions of all NCBs from Table 1 and CBs from the paper by Gazeas and Stepień (2008).

The reduced  $\chi^2$  values are equal to 7.12, 11.21 and 9.43 for total mass, AM and period, respectively. So high values of  $\chi^2$  mean a negligible probability that both samples were drawn from the same distribution.

We conclude that NCBs are in a different (presumably earlier) evolutionary stage than CBs. This leaves no candidates for a broken contact binaries predicted by TRO model but is in agreement with our model where CBs are individually in thermal equilibrium and stay in contact until component merging.

### 3.2. Period Variations

According to the TRO theory length of the orbital period of a CB varies in the course of a thermal oscillation. With a typical period of 0.3 d and a thermal time scale of  $10^6 - 10^7$  y we expect the time derivative of the period,  $dP/dt$ , to have typical values between a few times  $10^{-8}$  up to a few times  $10^{-7}$ , in units of d/y. Period variations of the same order are expected in the broken contact binary during the oscillation. Indeed, period variations of this order have been reported for several CBs and NCBs (Qian 2001ab, Zhu and Qian 2006, Zhu *et al.* 2009a, see also references to individual objects in Table 1). Can this be a proof for correctness of the TRO theory? Hardly, in our opinion. Here are some counterarguments.

Most of the individual values of period variations was determined from analysis of the  $O - C$  diagrams. A second order polynomial is fit to the observational data and a period variation is described by the quadratic term. Errors of the quadratic coefficient are seldom given by the author analyzing the  $O - C$  diagram but if the error is given, it is typically just a few times smaller than the coefficient itself. There are, however, several cases when no apparent trend is visible in the  $O - C$

diagram (see *e.g.*, Kreiner *et al.* 2001). These cases have usually been ignored when discussing period variations of a sample of variables. Obviously, such samples are strongly biased toward binaries with large period variations. Only recently, two systematic surveys of period variations of CBs have been carried out; Kubiak *et al.* (2006) analyzed 569 CBs observed by the OGLE team and Pilecki (2010) analyzed almost 6000 short-period eclipsing binaries (among them CBs, NCBs and detached binaries) observed within the ASAS program. Both analyzes showed that only about one third of the objects show measurable period variations. For the rest of stars any variations are below the accuracy of determination. Unfortunately, the uncertainties of both surveys were still quite considerable due to a short duration of OGLE and ASAS programs. Nonetheless, any statement about the presence and magnitude of secular period variations in CBs and NCBs seems premature and must await more systematic investigation.

In addition to any possible systematic trend,  $O - C$  diagrams of many binaries show a wavy behavior interpreted as periodic, or quasi-periodic orbital period variation. Two possible mechanisms are considered in respect to them: presence of a third body or variation of stellar moment(s) of inertia due to magnetic activity cycles, as suggested by Applegate (1992). Third body is usually preferred (Liao and Qian 2010) because almost all cool short-period binaries are expected to have such a companion (see Introduction). The resulting periods of outer orbits range from a few years up to the length of the time interval covered by  $O - C$  diagrams. Liao and Qian (2010) even claim a detection of a cyclic period variation of WW Dra with the length of 112 y. Because of the ubiquitous presence of third companions, interpretation of  $O - C$  diagrams of CBs and NCBs needs the utmost care as stressed by Hilditch (1989). Many apparently parabolic diagrams show a sudden turn when additional observations are added and instead of a secular period variation a cyclic variation appears (J. Kreiner – private communication). Lack of any secular period variation is reported for the following objects from Table 1: NP Aqr, DO Cas, V747 Cen, VV Cet, FS Lup, SW Lyn, V1374 Tau of SD1 type and IV Cas, YY Cet, AX Dra, FG Gem, SZ Her, DI Peg, HW Per, V1241 Tau and AW Vul of SD2 type. In most these cases a cyclic variation is present which is interpreted in terms of the tertiary system, although some authors also consider the Applegate mechanism (Zavala *et al.* 2002).

There exists yet another possible mechanism of period variations of CBs and NCBs. The energy transport between the components of a CB or SD1 binary requires about  $10^{-4} - 10^{-5} M_{\odot}/y$  of matter flowing back and forth between the components (Webbink 1977c, Martin and Davey 1995, Stepień 2009). Random fluctuations of this flow, resulting *e.g.*, from the variable magnetic activity, at the relative level of  $10^{-3}$ , will result in period variations of the order of  $10^{-7}$  d/y. Weaker fluctuations will produce, of course, correspondingly weak period variations. All these variations will have a random distribution. Kubiak *et al.* (2006) concluded that the observed distribution of period variations of CBs can, indeed, be described by the

Gaussian curve. This indicates a purely random mechanism for these variations.

We conclude that the observed period variations of CBs and NCBs can be caused by so many different mechanisms that their presence in individual binaries cannot be used as an evidence for the existence of TRO.

#### 4. Discussion

The problem of evolutionary connection between W UMa-type variables and other cool short-period binaries has been discussed since their discovery. Hilditch (1989) summarized his earlier investigations of several contact and semi-detached stars with the conclusion that there must exist two ways of forming cool CBs: first, when a moderate mass transfer from a primary makes a secondary also fill up quickly its Roche lobe, forming in effect a contact binary of W-type in a state of TRO, and second, when the mass transfer proceeds until mass ratio reversal so that an Algol-type configuration is first formed, which next transforms into a marginal-contact system and then into a deep-contact A-type system. Note, that the Kuiper paradox does not apply to the latter binaries, as was already mentioned in Section 1, so each component can individually be in thermal equilibrium and fit to its Roche lobe. There is no need for artificially inflated secondary (former primary) by elevated value of its specific entropy in this case, although energy must be transported to it from the present primary. If Hilditch was right, there would exist a fundamental difference in the physical properties of A-type and W-type variables. However, as we know now, the division between two types has no fundamental physical meaning and is most probably connected with the degree of spottiness of the primary component. As Gazeas and Niarchos (2006) demonstrated, the most massive CBs with periods longer than 0.5 d are only of A-type. Their primaries show very little or no magnetic activity and they are permanently hotter than their companions. The least massive CBs with periods shorter than 0.3 d are, on the other hand, only of W-type. Here, the primaries possess deep convection zones and are expected to be heavily spotted which decreases their mean surface temperatures. The secondaries are very likely much less spotted (Stepień *et al.* 2001). Both, A- and W-types occur for CBs with the intermediate masses and periods between 0.3 d and 0.5 d. Differences between the mean surface temperatures of both components (they decide about the classification of the variables as an A-type or W-type) are rather small among these binaries and change sometimes sign, which makes some binaries to alternate between the types.

The idea of an evolutionary sequence of cool CBs: detached  $\rightarrow$  SD1  $\rightarrow$  SD2  $\rightarrow$  CB was later suggested by Shaw (1994). Accurate stellar parameters were then available for very few NCBs so the author based his proposition on the apparent similarity of SD2 type stars to the A-type CB. He noted that a moderate shortening of the period would bring an SD2 binary into contact and with the efficient energy transport between the components it would simply become an A-type CB. He

also noted that a typical light curve of an SD1 variable shows asymmetry resulting presumably from a hot spot on the trailing side of the cool component due to the mass transfer through the inner Lagrangian point from the hot component. He concluded that SD1 stars are nearer the beginning of their evolution toward CBs and after mass ratio inversion they become SD2 stars. We confirm here his proposition quantitatively with the accurately known stellar parameters of sufficiently numerous samples of cool close binary stars, as shown in Section 3.

The problem of an evolutionary status of NCBs was also considered by Yakut and Eggleton (2005). Based on available at that time observations they determined stellar parameters of several cool close binaries of different types, compared them with each other and with evolutionary models obtained earlier by Eggleton and collaborators. In particular, they noticed that the total mass and angular momentum of NCBs are significantly higher than of CBs. However, they tried to explain this discrepancy within the TRO model by making an arbitrary assumption that massive CBs with long periods spend more time in the broken contact phase (identified with the SD1 configuration) as opposite to low mass CBs which must spend very little (if any) time in the broken contact phase because so few SD1 binaries is observed with short periods. Yakut and Eggleton (2005) also discuss evolutionary status of SD2 binaries. Following Hilditch (1989) they suggest that these are binaries past case A mass transfer, *i.e.*, with a primary filling the Roche lobe when it is still on MS. In other words, SD2 binaries are presently in the Algol configuration. But what about early phases of mass transfer in these binaries, before mass ratio reversal? They argue that some of the observed SD1 binaries are in fact “first-timers”. Because, however, this phase takes a much shorter time according to a standard mass transfer model (Webbink 1976, Nakamura 1985, see also above) we should observe several times more SD2 than SD1 binaries. The observations show otherwise as we know and as also Yakut and Eggleton noted; both groups of stars are roughly equally numerous. The authors concluded that the majority of SD1 binaries must be in a broken contact phase of TRO. Unfortunately, they could not distinguish between these two groups of SD1 binaries and were unable to offer any observational test to do so. This apparent ambiguity in evolutionary status of SD1 binaries leads authors analyzing individual objects of this type to the standard conclusion in a form of a rhetoric question: is the analyzed variable a first-timer or a CB in a broken contact phase (see references to Table 1). With our model of mass transfer described in Section 3, all SD1 binaries are first-timers and they should be as numerous as SD2 stars. As mentioned earlier, a short phase between SD1 and SD2 configuration exists when both components have almost equal masses and are in contact. FT UMa may be an example of such CB with a total mass less than  $3 M_{\odot}$  (Yuan 2011). The binary has a period of 0.65 d, component masses of  $1.49 M_{\odot}$  and  $1.46 M_{\odot}$  ( $q = 0.98$ ), and radii of  $1.79 R_{\odot}$  and  $1.78 R_{\odot}$ . The existing observational data do not allow to decide whether the binary is still prior to, or just past mass reversal.

New data on CBs and NCBs obtained in the recent years, based on systematic and accurate photometric and spectroscopic surveys of cool close binaries like ASAS, OGLE and DDO, did not resolve problems inherent in the TRO model. On the contrary, the problems seem to be even more severe than ever. Adoption of the idea that all cool CBs are past mass ratio reversal, *i.e.*, they are in a similar evolutionary state as short-period normal Algols, explains all important observational facts in a consistent way. CBs form a low-mass, short-period end of a sequence of binaries showing so called Algol paradox (when the less massive component is evolutionary more advanced than its more massive companion). Both components of such configuration are in thermal equilibrium. In addition, a common envelope surrounding both components of a CB enforces circulation carrying energy from the more to less massive component, which results in an apparent equality of the average surface brightness.

The model presented in this paper can be verified by direct hydrodynamical simulations of the mass transfer in CBs and NCBs. Two-dimensional computations of a mass transfer in a CB carried out by Martin and Davey (1995) demonstrated the existence of an asymmetric stream deflected by the Coriolis force and encircling the secondary component, but two dimensions restricted severely dynamics of the flows in the common envelope. Full 3D calculations are necessary, as shown by Oka *et al.* (2002). It is apparent from their results that dynamics of matter in surface layers of a donor is dominated by the Coriolis force just as is dynamics of air in the Earth atmosphere. Including an accretor, first under-filling and then overfilling its critical Roche lobe, into the computational domain would greatly contribute to our understanding of the mass and energy exchange between the components of NCBs and CBs.

With fast developing techniques for measuring velocity fields over the stellar surface we may soon be able to obtain detailed maps of mass motions in NCBs and CBs. Approximate information can already be obtained from the analysis of the spectral line profiles as was done *e.g.*, by Rucinski and his collaborators. As an example, the broadening function of AW UMa indicates that the radial velocity of the polar regions in both components is lower than, but in the equatorial regions is close to velocity expected for stars filling their Roche lobes and rotating synchronously with the orbital period (Pribulla and Rucinski 2008). This peculiar velocity field can be explained by our model (Stępień 2009). Another model prediction concerns the temperature distribution over the surface of a low mass component in CBs and SD1 binaries with a hotter equatorial belt and cooler polar caps. The accurate temperature mapping should be able to verify this prediction.

Finally, we stress that if the future observations confirm the existence of the equatorial bulge in CBs and SD1 binaries, their conventional model assuming strict hydrostatic equilibrium and stellar surfaces lying exactly on equipotential surfaces must be abandoned. Dynamical effects will have to be included when modeling these stars.



## 5. Conclusions

The observed properties of cool close binaries with one component filling, and the other nearly filling its Roche lobe (called NCBs) fit to the recent model of origin and evolutionary status of cool CBs developed by one of us (Stępień 2006ab, 2009, 2011a, Gazeas and Stępień 2008, Stępień and Gazeas 2012). An important starting point of the model is based on the observational, as well as theoretical, arguments that the orbital period distribution of young cool binaries is concentrated around 2–3 d. This fact is supplemented with an assumption that components of these binaries possess magnetic winds carrying mass and AM in the same way as single stars of the lower MS. Mass loss is rather moderate (of the order of  $0.1 M_{\odot}$  during the whole MS age) so it influences little a nuclear time scale of either component. But, as detailed computations of the evolution of cool close binaries show, the AM loss time scale is close to MS life time of a primary component for typical values of the initial orbital period (Stępień 2011a). In effect, the typical primary reaches RLOF when it is close to TAMS. Following RLOF, a mass transfer occurs. With dynamical effects taken into account, it can be shown that, until mass reversal, the process takes time comparable to the thermal time scale of the secondary component, as opposed to the conventional models of mass transfer neglecting dynamics of the transferred matter. SD1 variables are identified with this phase of mass transfer. After mass ratio reversal a CB, or SD2 binary is formed, depending on the amount of AM left in the system. Further evolution of an SD2 binary is affected by two factors acting in the opposite direction: mass transfer from the present secondary to the present primary makes period increase whereas the AML makes it shorten. Depending on the relative importance of these processes the binary evolves either into contact or becomes an Algol (Stępień 2011b).

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